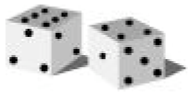


Activity 8 The Gambler's Fallacy

Objectives

- Apply the probability formula
- Compute combined probability
- Define *dependent* and *independent* events
- Determine the probability of two or more events
- Define the *gambler's fallacy* and relate it to mathematical probability

Materials	paper, pencils, calculators
Time	30 minutes
Math Idea	The belief that because a certain event has *not* occurred many times in a row that is more likely to occur the next time is known as the *gambler's fallacy.*

Prior Understanding

Students should know how to convert among fractions, decimals, and percents. They should also know how to find simple theoretical probabilities using the probability formula.

Introduction: Gambling Connection

You can use or adapt the following scenario as an introduction to the problem. Then have students do the activity and discuss the solution.

Sasha has three younger brothers, and her mother is about to give birth again. She is betting her brothers that this time her mother will have a girl. Assuming the chances of being born a boy and the chances of being born a girl are the same, how certain is Sasha of winning her bet?

Discussion

Students should realize that Sasha is a victim of the *gambler's fallacy.* She is confusing a single event with a sequence of four events and is mistaking **independent events** for **dependent events.** An outcome of boys on the three previous births does not remove "boy" from the pool of possible outcomes for the next birth, nor does it guarantee "girl" as the only possibility. No matter what has happened before, the probability of her mother giving birth to a girl (or a boy) at any one time remains 50%.

Exercise 1

Have students list the possible outcomes for tossing a coin four times. Then have them determine the probability of getting four tails in a row.

Have students list the possible outcomes for tossing a coin the fourth time after three tails have already come up. Then have them determine the probability that tails will come up again.

Have students compare these two problems, identifying any similarities and differences.

Discussion

When determining the **probability** of two or more events, it is important to know (a) whether you are determining the **probability** of a single event or the **probability** of a group of events, and (b) whether the outcomes of the events are **independent** or **dependent**.

The tossing of four coins in a row consists of *four events* with a total of 16 possible outcomes: HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, TTTH, TTTH, TTHT, TTHH, THTT, THTH, THHT, THHH. Only one of these outcomes shows four tails, so the **probability** of getting four tails is 1/16. The four tosses are ***independent***—the outcome of the first toss does not affect the outcome of the second, third, or fourth in any way. On each toss, the **probability** of getting tails (T) is 1/2, so the combined **probability** of getting four tails is $1/2 \times 1/2 \times 1/2 \times 1/2 = 1/16$ or 6.25%.

If three tails have come up, the tossing of the next coin is a ***single event*** whose outcome is ***independent*** of what has already happened. The **probability** of getting tails again is still 1/2 or 50%.

Here it does not matter what the **probability** of getting four heads in a row is: at this point, we are only dealing with the *next* toss. Students should note that although determining the **probability** of an outcome for a *group* of coin tosses is different from determining the **probability** of an outcome for *one* coin toss, the individual outcomes are **independent**.

Exercise 2

Have students determine the probability of drawing two aces in a row from a well-shuffled, complete deck of cards, if the drawn cards are replaced and the deck is shuffled again after each draw.

Have students determine the probability of drawing two aces in a row from a well-shuffled, complete deck of cards, if the drawn cards are not replaced and the deck is shuffled again after each draw.

Have students compare these 2 problems, identifying any similarities and differences.

Discussion

The drawing of two aces in a row consists of two events. When the drawn cards are returned to the deck, the two events are **independent events**. Each time there are 4 aces out of 52 cards to choose from, so the **combined probability** is $4/52 \times 4/52 = (1/13)^2 = 1/169$ or about 0.59%. Students should note that it is possible to draw the same ace twice.

When the drawn cards are *not* returned to the deck, the two events are ***dependent events***— the result of the first draw affects the potential outcome of the second draw. If the first draw is an ace, there are 3 aces left out of 51 cards in the deck. The **probability** of drawing another ace is now 3/51 and the **combined probability** is $1/13 \times 3/51 = 1/221$ or about 0.45%.

Activity 8 The Gambler’s Fallacy Teacher Support

Vocabulary

probability a number from 0 to 1 that expresses the likelihood that a given event (or set of outcomes) will occur

combined probability the probability that two or more events will all occur

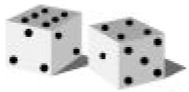
independent events events in which the outcome of the first event *does not* affect the outcome of subsequent events

dependent events events in which the outcome of the first event *does* affect the outcome of subsequent events

Ongoing Assessment

What is the probability of drawing four clubs in a row from a well-shuffled, complete deck of cards if the drawn cards are not replaced?

($13/52 \times 12/51 \times 11/50 \times 10/49 = 0.00264$)



Added Practice 8 The Gambler's Fallacy

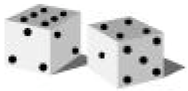
Name ______________________________Date______________________

1. Suppose you have already drawn two cards from a well-shuffled, complete deck and neither one is a five. What is the probability that you will draw a five the next time if the first two cards are not replaced?

2. Suppose you have already drawn a four and a five from a well-shuffled, complete deck. What are the chances of drawing a five the next time if the first two cards are not replaced?

3. A friend of yours has a cousin who was in a minor plane crash and survived. Whenever your friend has to fly in an airplane, she insists on being accompanied by her cousin, figuring that the probability of her cousin being in two plane crashes is very small. Is your friend's reasoning correct? Why or why not?

4. An acquaintance of yours is concerned about a recent terrorist bombing of an airline flight. He decides that on all future flights he takes, he will bring a bomb in his suitcase. He reasons that the probability of two bombs being on a plane is very, very low. Will his plan decrease his chances of being killed by a terrorist bombing while flying? Why or why not?



Answer Key Added Practice 8 The Gambler's Fallacy

1. On the next draw there are 50 cards left in the deck, 4 of which are fives. The probability is 4/50 = 8%.
2. On the next draw there are 50 cards left in the deck, only 3 of which are fives. The probability is 3/50 = 6%.
3. Your friend's reasoning is not correct. Plane flights are not dependent events—the outcome of one flight does not have an effect on the outcome of subsequent flights. Assuming that the probability of a commercial flight crashing is one in a million, a person will be subjected to that one-in-a-million risk each time he or she takes a commercial flight, whether or not he or she has been in a previous crash. In reality, there are specific causes of plane crashes: faulty or damaged airplane parts, engine failures, weather conditions, pilot incompetence, etc. However, because you cannot evaluate all of the variables each time you take a flight, you have to assume that plane crashes are more or less random occurrences. That is, because the factors leading to a crash are unknown and unpredictable before the crash, you have to approach crashes as rare chance outcomes. Clearly, the presence or absence of someone who has been in a previous crash does not influence the outcome of the flight in any way.
4. Your friend's plan will not change his chances of being blown up by a terrorist. A terrorist choosing to blow up the plane that your friend is on is a random occurrence. Let's say, hypothetically, that the chance of this happening on any particular plane is 1/5,000,000. If your friend knows that he is carrying a bomb on a particular flight, then he can be 100% confident that there is at least one bomb on his plane. The combined probability of his bomb *and* a terrorist's being on his plane is the product of the individual probabilities, that is 1 x 1/5,000,000 = 1/5,000,000. Although it is true that the probability of two people putting a bomb on the same plane independently is very, very small, what your friend is carrying in his suitcase has no effect on the situation.