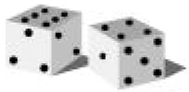


Activity 5 Shared Birthdays

Objectives

- Understand concept of probability
- Conduct trials and observe outcomes
- Apply probability formula
- Differentiate between experimental and theoretical probabilities

Materials	pencils, paper, calculators
Time	5–10 minutes on one day to collect data
	30–40 minutes on the next day to analyze data
Math Idea	In any group of 23 randomly chosen people, there is about a 50% chance that two or more people in that group will have the same birthday.

Prior Understanding

Students should know how to convert among fractions (ratios), decimals, and percents. They should also know how to find simple theoretical probabilities using the probability formula.

Introduction: Gambling Connection

You can use or adapt the following scenario as an introduction to the problem. Then have students do the activity before revealing the answer and its explanation:

> *Sam notices there are 23 people with their dogs at the pet show. He bets his friend Rick that no two people in the group have the same birthday. Who is more likely to win the bet—Sam or Rick?*

Discussion

First have students evaluate whether Sam or Rick is more likely to win the bet, based on their experimental results. *(Students' evaluations will depend on the results they obtained.)* Then have them decide based on the theoretical probability. *(It is slightly more likely that Sam will lose and Rick will win.)*

Exercise

Have several students be data collectors in a survey of about 100 students. Have data collectors ask each student to write his or her birthday (month and day) on a slip of paper. Then have them collect the slips and bring them to class.

Pool all the slips of paper and randomly divide them into groups of 23. Divide the class into small groups, giving each group a set of 23 birthday slips. Have group members go through their slips one at a time until they get a match. Students can use a chart like the one shown to keep track of the birthdays.

Slip	Birthday
1	May 3
2	December 15
3	
4	
5	

Next, Record the number of sets that have matches. As a class determine what percent of the sets have shared birthdays.

$$\frac{\text{number of groups with matches}}{\text{total number of groups}} \times 100 = \%\text{with shared birthdays}$$

Discussion

Discuss the results with the class. If their results were not close to 50%, ask students to account for the difference. Results will vary depending on the number of students surveyed; if there are only two or three sets of 23 students, the percent is likely to be less than 50%; if there are more sets of 23 students, the percent is likely to be closer to 50%.

Be sure students understand that the activity statement expresses **a theoretical probability**, whereas their results represent an **experimental probability**. Explain that this activity is similar to tossing a fair coin. Just as there is a 50% chance of getting heads and a 50% chance of getting tails on any one toss, there is about a 50% chance of having a shared birthday and about a 50% chance of not having a shared birthday in any one group of 23 people. You can demonstrate this point by having students toss a coin once for each set of 23 birthdays. Then calculate the percent of tosses that came up heads. If there are only three tosses (three sets of 23), heads might only come up once (33.33%); if there are 10 tosses (ten sets of 23), heads might come up 4 or 6 times (40% or 66.67%); if there are 100 tosses, heads might come up 49 times (49%); and so on. The larger the number of trials (the more sets of 23), the closer the **experimental probability** will be to the **theoretical probability** of about 50/50.

To show students how to obtain the **theoretical probability**, have them consider a group of 23 people lined up in a row. Once they know the first person's birthday, the **probability** that the second person's birthday is different from the first's would be 364/365 (exclusive of leap year). The **probability** that the third person's birthday is different from both the first and second's is 363/365. The **probability** that the fourth person's birthday is different from the other three's is 362/365. Suppose the pattern continues until you reach the 23rd person. The expression

$364/365 \times 363/365 \times 362/365 \times \ldots \times 343/365$ represents the probability that all 23 people have a different birthday. Let students use calculators to find the product (0.492702765676014592774582771662967) and change it to a percent (≈49%). Since this is the probability that *no two* people in the group have the same birthday, the probability that *at least two* people have the same birthday is ≈51%.

Activity 5 Shared Birthdays Teacher Support

Vocabulary

probability a number from 0 to 1 that expresses the likelihood that a given event will occur

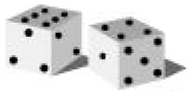
experimental probability probability determined by conducting a series of tests or trials and observing the number of favorable results compared to the total number of trials

theoretical probability probability determined by comparing the number of ways a favorable result can happen to the total number of equally likely possible outcomes

mathematical notation meaning about or approximately equal to

Ongoing Assessment

Have students speculate about the chances that two people in a group of 30 have the same birthday. Divide the birthday slips into groups of 30 and have students repeat the activity to determine an experimental probability for this situation. Then let students calculate the theoretical probability and compare it to their results. *(Students' predictions should show a higher expectation than 50%. Experimental results will vary. The theoretical probability is about 71%.)*



Added Practice 5 Shared Birthdays

Name ______________________________ Date______________________

1. In a group of 40 people, how likely is it that two or more people have the same birthday? Make a prediction, then complete the activity. You may wish to use a separate sheet of paper.

   (a) Make a list of 40 people whose birthdays you can look up in an encyclopedia or other reference book. You might choose presidents; astronauts; sports figures; movie, television, or music personalities; explorers; scientists; and so on. Look up and record their birthdays. When you get a match, you can stop; or you can continue to see whether more than two people on your list share a birthday.

   (b) Combine your results with those of your classmates to calculate the percent of the time there are shared birthdays in a group of 40 people.

   $$\frac{\text{number of groups with matches}}{\text{total number of groups}} \times 100 = \%\text{ with shared birthdays}$$

   (c) Use a calculator to determine the theoretical probability that two or more people in a group of 40 share a birthday. Compare this result to the experimental probability you obtained.

2. Within any group of 367 people, what is the likelihood that at least two will have the same birthday? Explain your answer.

3. In a group of 20 telephone numbers selected at random, what is the likelihood that two or more numbers match in the last two digits?

Answer Key Added Practice 5 Shared Birthdays

1. Students' predictions should indicate an expectation higher than 71% based on their previous experiences with this problem. The theoretical likelihood for a group of 40 people is about 89%.
2. 100%; There are 366 possible birthdays (365 days plus February 29 during a leap year), so at least two people must share a birthday. A good analogy is placing 367 letters in 366 mailboxes. Each mailbox gets one letter and one mailbox must get two letters.
3. Once the first telephone number is picked, the probability that the last two digits of second phone number **do not** match the first is 99/100; the probability that the last two digits of the third phone number do not match either the first or second is 98/100; and so on. Then $99/100 \times 98/100 \times 97/100 \times \ldots \times 81/100 \approx 0.15902 \approx 16\%$ represents the probability that the last two digits of all 20 phone numbers do not match. That means there is about an 84% chance that the last two digits will match.