Activity 4 Heads or Tails?

Objectives

- Understand concept of probability
- Understand concept of randomness
- Conduct trials and observe outcomes
- Relate theoretical probabilities to actual situations and experimental probabilities

Materials	coins, paper, pencils, calculators
Time	30–45 minutes
Math Idea	(A) When flipping a coin, the probability that the coin lands heads up is the same as the probability that the coin lands tails up—1/2 or 50%. (B) When flipping two coins, the probability that the coins show the same side (2 H or 2 T) is the same as the probability that the coins show different sides (1 H, 1 T)—1/2 or 50%.

Prior Understanding

Students should know how to convert among fractions (ratios), decimals, and percents. They should also know how to find simple theoretical probabilities.

Introduction: Gambling Connection

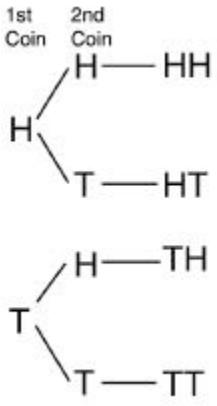
You can use or adapt the following scenario as an introduction to the problem. Then have students do the activity before revealing the answer and its explanation.

Two friends of yours are betting on the outcome of coin tosses. One of them always likes to bet on tails, and tails has come up three times in a row. She decides to change her bet for the next toss. She reasons that after three tails, a heads is "due." What is your advice?

Discussion

(A) Have students answer the question based on their experimental results. (Although responses will vary, students should recognize that whether or not the friend changes her bet, she can't improve her chances of winning.) Stress that the **outcome** of any one trial (or a small group of trials) cannot be predicted with certainty. The **outcome** of coin tosses is always **random** and the probability for heads is the same as the probability for tails. Previous **outcomes** do not affect current or future **outcomes** in any way. No matter how many times in a row a coin lands tails up, there is still only a 50% chance that it will land heads up on the next flip.

(B) Students can also draw a **tree diagram** to determine **possible outcomes**. To help students understand that H T is a different **outcome** from T H, have them think about it as flipping two different coins, such as a dime and a penny.



Using this method, students should note that there are 4 **possible outcomes**, each with a probability of 1/4 or 25%. Using **theoretical probabilities** with two events, as is the case here, the probability that both will occur is the product of the two separate probabilities. For each coin there is a 50% chance of landing heads up, so the **theoretical probability** of both being heads (or tails) is .50 × .50 = .25 or 25%. In 2 of the 4 **outcomes**, the same side is showing, so the probability is 2/4 = 1/2 = 50%. Similarly, there are 2 out of 4 **outcomes** with different sides showing.

Exercise 1

Divide students into pairs. Have partners take turns flipping a coin 20 times. Using H for heads and T for tails, to keep track of the results, have students record the number and percentage of times heads is followed by tails, and so on. Have them count the total number of heads and tails, and calculate the percent of heads and percent of tails.

$$\%\,\text{heads} = \left(\frac{\#\,\text{heads}}{20} \times 100\right) \qquad \%\,\text{tails} = \left(\frac{\#\,\text{tails}}{20} \times 100\right)$$

Exercise 2

Create a class chart on the chalkboard like the one shown. Have each pair record their results. As a class, find the total number of heads, total number of tails, and total number of flips. Then calculate the percent heads and percent tails for the total.

Pairs	Heads	Tails	% Heads	% Tails
Joe, Marie	7	13	35%	65%

Discussion

Have students compare the class percent with the percents obtained by student pairs. Discuss which results are closer to 50% and ask students to explain why. Generally, the percents obtained by student pairs will vary widely from 50% and the class percent should be closer to 50%, although some student pairs may obtain a 50% result.

Exercise 3

Without giving students coins, have them determine all the possible outcomes for flipping two coins. Ask students to calculate the theoretical probability for each outcome and the theoretical probability that both coins land with the same side up.

Discussion

Tossing a coin is a **random process**, with heads and tails being equally likely **outcomes**. With one coin, of the 2 possible **outcomes** only 1 is favorable (either heads or tails), so the **theoretical probability for each coin** is 1/2 or 50%. For two coins, there are four possible outcomes (i.e., heads/heads, tails/tails, heads/tails, tails/heads). The theoretical probability of each outcome is ¼ = .25. Students may not understand initially that heads/tails is a different outcome than tails/heads. As the number of trials increases, the **experimental probability** comes closer to the **theoretical probability**.

Activity 4 Heads or Tails? Teacher Support

Vocabulary

experimental probability probability determined by conducting a series of tests or trials and observing the number of favorable results compared to the total number of trials

outcome any possible result of an experiment or activity

probability a number from 0 to 1 that expresses the likelihood that a given event (or set of outcomes) will occur

random process process in which each possible outcome has the same chance, of occurring; that is, all possible outcomes are equally likely

theoretical probability probability determined by comparing the number of ways a favorable result can happen to the total number of equally likely possible outcomes

tree diagram a diagram used to show the total number of possible outcomes

Ongoing Assessment

Let students experiment with flipping three coins and calculating the probability that all three land with the same side up. Then have them find the theoretical probability by listing all the possible outcomes. *(With three coins, there are 8 possible outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT. In 2 of the 8 outcomes, the same side is showing, so the probability is 2/8 = 1/4 = 25%.)*

Added Practice 4 Heads or Tails?

Name _____ **Date** _____

1. List all the possible outcomes when you roll one die. Are all the outcomes equally likely? Explain.

2. Find the theoretical probability for each of the following events when rolling one die. Express your answers in fraction and percent form.

   (a) rolling a six

   (b) rolling either a four or a five

   (c) rolling an even number

   (d) rolling an odd number

3. Roll a die 30 times. Record the outcome of each roll.

   (a) Based on your results, find the theoretical probability for each outcome.

   (b) Combine your results with those of your classmates. Find the same probabilities using the totals for your class.

   (c) Compare the experimental probabilities you obtained with the theoretical probabilities you found.

4. List the possible outcomes when you roll two dice. Find the probability of rolling a 2 on both dice.

5. From a complete deck of shuffled cards, what is the probability of drawing a queen? What is the probability of drawing the queen of hearts?

Answer Key Added Practice 4 Heads or Tails?

1. Possible outcomes: 1, 2, 3, 4, 5, 6; There are six sides numbered 1 through 6; all outcomes are equally likely as long as the die is fair.
2. (a) probability of rolling a six is 1/6 or $\approx$ 16.67%

   (b) There are two favorable outcomes out of six possible outcomes, so the probability of rolling either a four or a five is 2/6 = 1/3 or $\approx$ 33.33%.

   (c) and (d) There are three even numbers on a die (2, 4, 6) and three odd numbers (1, 3, 5), so the probability of rolling an even number is 3/6 = 1/2 = 50% and the probability of rolling an odd number is 3/6 = 1/2 = 50%.
3. The probabilities obtained using the class totals are likely to be closer to the theoretical probabilities than those obtained by individual students.
4. There are 36 possible outcomes. They can be represented as ordered pairs: (1, 1), (1, 2), (1, 3), . . . , (6, 1), (6, 2), . . . , (6, 6). Students should realize that (1, 3) and (3, 1), for example, are two different outcomes. Of the 36 outcomes, only one shows 2 on both dice, so the probability is 1/36 $\approx$ 2.78%. Alternatively, the probability of rolling 2 on the first die is 1/6; the probability of rolling 2 on the second die is 1/6, so the probability of both happening is 1/6 $\times$ 1/6 = 1/36.
5. There are 4 queens in a deck of 52 cards, so the probability of drawing a queen is 4/52 = 1/13 $\approx$ 7.69%. There is only one queen of hearts, so the probability is 1/52 $\approx$ 1.92%.