



Activity 10 Red and Black Marbles and the Lottery

Objectives

- Apply the probability formula
- Differentiate between *dependent* and *independent* events
- Explain the effect of increasing or decreasing the size of a pool of numbers on chance

Materials	paper, pencils
Time	20–30 minutes
Math Idea	Once a specific combination of numbers is chosen, the composition of various groups within a pool of numbers becomes irrelevant.

Prior Understanding

Students should know how to convert among fractions, decimals, and percents, as well as find probabilities using the probability formula.

Introduction: Gambling Connection

You may wish to read aloud the following to begin a classroom discussion and introduce this activity.



A lottery player might reason that because there are more spread-out combinations (such as 3, 16, 24, 30, 37, 42) than strings of consecutive numbers (such as 34, 35, 36, 37, 38, 39) or numbers that are below a certain value (such as 1, 3, 4, 6, 8, 9), he or she can increase the chances of winning by selecting numbers that are spread out over the entire range of possible numbers.

Discussion

While it is true that there are more collections of numbers spread out than there are collections of numbers under ten or collections of consecutive numbers, that is irrelevant as far as random selection is concerned. The prize is not given for matching the *pattern* of the winning numbers.

Once all the items in the pool are viewed as individual choices (as opposed to members of a group), it becomes clear that each choice has the same chance of being selected as any other choice. Although people tend to group their number choices together according to some characteristic that they think the numbers have in common, when it comes to random selection, none of the number combinations has anything in common with any other number combination: each one is an individual combination out of millions of combinations.



Example 1

Give students the following three scenarios and have them determine the corresponding probabilities.

1. A bag that contains five red marbles and five black marbles; one of the red marbles is marked with an **X**. You reach into the bag without looking and pull out a marble at random. What is the probability of choosing a black marble? What is the probability of choosing the marble with the **X** on it?

2. You remove from the bag all of the red marbles except the one marked with the **X** and put in four more black marbles. What is the probability of choosing a black marble? What is the probability of choosing the marble with the **X** on it?

3. You remove all but one of the black marbles and put in 8 more red marbles. What is the probability of choosing a black marble? What is the probability of choosing the one with the **X** on it?

Discuss the effect of choosing different collections of numbers on winning the lottery.

Discussion

For 5 red and 5 black marbles, $P(\text{black marble}) = 5/10 = 1/2$ or 50%; $P(\text{marble X}) = 1/10$ or 10%. For 1 red and 9 black marbles, $P(\text{black marble}) = 9/10$ or 90%; $P(\text{marble X}) = 1/10$ or 10%. For 9 red and 1 black marble, $P(\text{black marble}) = 1/10$ or 10%; $P(\text{marble X}) = 1/10$ or 10%. Students should notice that the **probability** of choosing



the red marble with the **X** is the same regardless of the compositions of the remaining red and black marbles in the bag. That is, the **probability** of picking a *specific* red marble (which is different from picking *any* red marble) is the same whether 90% of the marbles are red or only 10% of the marbles are red.



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Teacher Support

Vocabulary

probability a number from 0 to 1 that expresses the likelihood that a given event (or set of outcomes) will occur

independent events events in which the outcome of the first event *does not* affect the outcome of subsequent events

dependent events events in which the outcome of the first event *does* affect the outcome of subsequent events

Ongoing Assessment

Angela always picks the numbers 4, 8, 15, 17, 23, and 26 for the lottery. While she is away on vacation, her combination is picked. Upon returning, she decides to change her six number combination since it has already been picked. Angela believes that her number combination is less likely to be picked because it was picked only a few days ago. Is she correct? Why or why not? *(In reality this is not a bet that two lottery drawings in a row will both be the given numbers. It is only a bet that the next drawing will be the given numbers. Like the coin tosses discussed in the Head or Tails? and The Gambler's Fallacy Activities, earlier lottery drawings are independent events: the probability of any one outcome is always the same, regardless of what has happened before. The probability of the same numbers winning given that they won the last time, is $1 \times 1/12,271,512 = 1/12,271,512$.)* [See page 98 for additional discussion]



Added Practice 10 Red and Black Marbles and the Lottery

Name _____ Date _____

1. Michael plans to use birth dates to select his lottery numbers. Then he remembers that birth dates never go past 31, and the lottery uses numbers 1 through 48. Will Michael hurt his chances of winning if he decides to use birth dates? Explain your answer.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	?	?	?	?
?	?	?	?	?	?	?
?	?	?	?	?	?	?

2. Erin wants to pick the numbers 1, 2, 3, 4, 5 and 6, but her friend tells her that this is not a good idea because the probability of six consecutive numbers being drawn is so small. Would Erin improve her chance of winning if she chose a more varied six-number combination? Explain your answer.

3. John's sister wins the lottery. She advises him to stop playing the lottery for awhile. She reasons that his chances of winning the lottery have decreased, since the likelihood that two people in the same family would win the lottery is so small. Is she correct? Why or why not?



4. Read the advertisement, then answer each question.

WIN THE LOTTERY IN 18 DAYS OR LESS GUARANTEED!!!
STEVE PLAYERS LOTTERY POWER WORKOUT

There are over 20,000 copies of this book in print, and for a very good reason:
THIS SYSTEM WORKS!!!

It will only cost you \$5.00 each day to buy the tickets and our guarantee says the rest:
Either you win your state's lottery game in the first 18 days that you use the system or we
will gladly refund the full purchase price of the system.

NO FINE PRINT!!!

DON'T WAIT - ORDER TODAY - ONLY \$35. - / CATALOG \$2.00

Name: _____ Address: _____

City: _____ St: _____ Zip: _____

- (a) Although the advertisement states that there is "no fine print," can you think of what the "catch" might be?
- (b) How much will it cost you if you buy the tickets for the full 18 days?
- (c) Do you think Steve Players is making money from this business?
- (d) Would it be a wise investment of your money to buy this "system"?



Answer Key Added Practice 10 Red and Black Marbles and the Lottery

1. and 2. Neither Michael nor Erin will improve or hurt their chances of winning. Of the 12.3 million possible 6-number combinations, there are more combinations that have numbers spread out over the entire range than there are combinations that have consecutive numbers or combinations that have numbers all less than 32. So the probability that the winning combination will have spread-out numbers is higher than the probability that the winning combination will have all consecutive numbers, or the probability that the winning combination will have numbers all less than 32. However, that is not what players are betting on in a lottery. They are only allowed to bet that the one 6-number combination they chose will be the one that is randomly selected. Knowing that the winning combination is more likely to be “spread out” does not increase their chances of guessing the right combination. The following analogy may make this concept more understandable: Suppose there are 100 people in your neighborhood and one person was going to be chosen at random. Your neighborhood is 60% female and 40% male. If you are betting on what the selected person’s gender will be, then you would bet female, since there is a 60% chance that the person will be female and a 40% chance that the person will be male. However, if you were betting that a specific individual would be picked (for example, your mother), the person’s gender would have no effect on their chances of being picked. One person is being selected randomly, so each person has an equal chance of being selected.

3. John’s sister is wrong. The outcomes of lottery drawings are not dependent events. Each outcome is random, and previous outcomes do not have any effect on future outcomes. The random number generator from which the numbers are chosen does not know who John is related to, where he lives, or any other information about him. It is a machine, and it simply draws numbers. John’s chances of winning



the lottery are the same before or after his sister, or any other person, wins the lottery.

4. (a) Some people who see this advertisement may believe that Steve Players is simply trying to trick customers through false statements and deceit, and that there is no truth to the claims made by the advertisement. Others may believe that the system actually works, and that they will win the lottery if they buy the system and use it. Most likely, both of these interpretations are incorrect. There is likely some degree of truth to the advertiser's claim. The book that you would receive if you sent in your money most likely states that you are guaranteed to win *some* lottery prize (for example, a \$1 scratch ticket prize or matching 1 out of 4 on the daily numbers). When the advertiser guarantees that you will "win your state's lottery game," he is probably not referring to the lottery jackpot.

(b) If you buy the tickets for the full 18 days, you will spend $\$5 \times 18 = \90 . In addition, the book costs \$35, so your total expenses would be $\$90 + \$35 = \$125$.

(c) Steve Players is most likely making money from this business. Many of those who buy the system would probably win some small prize as a result of all of the lottery tickets they had to buy. However, the large majority of these "winners" will be losing money since the chances of winning a \$1 prize or a \$5 prize are far greater than the chances of winning a prize of \$125 (to break even), or a prize of more than \$125 (to make a profit). Most likely, anyone who won any prize, no matter how small, would be ineligible to get a refund. In addition, the majority of those who won nothing probably would not demand a refund due to embarrassment over being duped.

(d) This system would not be a wise investment of your money, since the large majority of people who use this system probably lose money.